COL874
Advanced Compiler Techniques

Modules 141-145

Presented by: Jai Arora

Lec-141: Affine Space Partitions

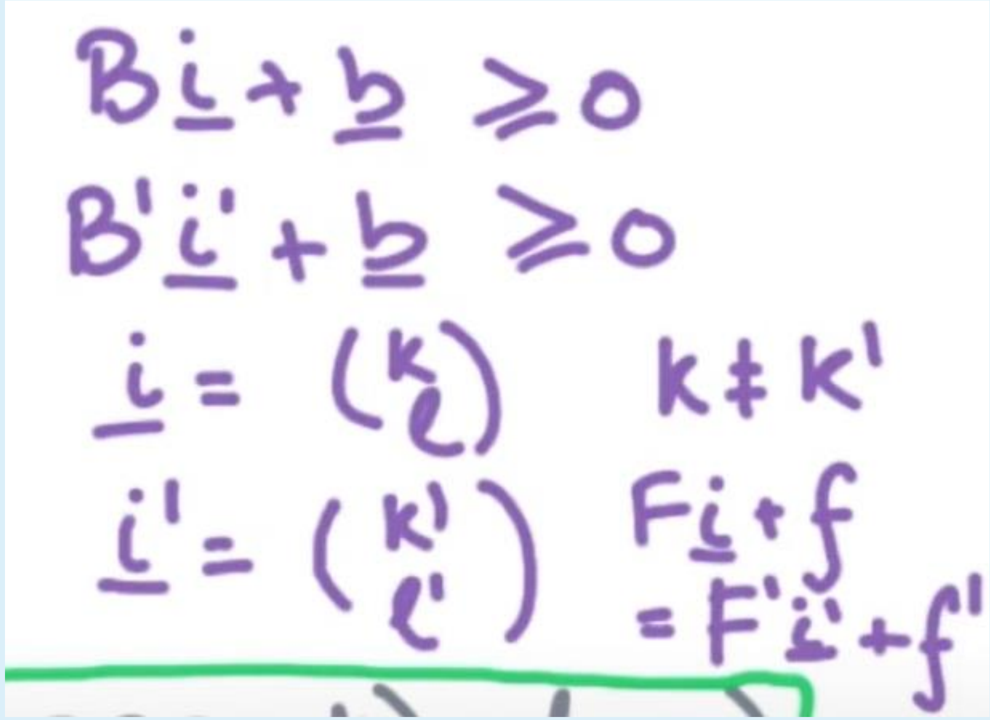
```
for(i = 0; i < 1000; i++)
  for(j = 0; j < 1000; j++)
    A[i, j] = A[i+1, j-1]
```

→

```
par for(k = 0; k <= 1998; k++)
      for(l = 0; l <= min(k, 1999 - k); l++)
        A[k - l, l] = A[k - l + 1, l - 1]
```

- Data dependencies exist across different iterations of the first loop
- Possible to transform the axes of the program to exploit **Synchronization Free Parallelization**
- Iterations of the outer loop in the transformed program can be done in parallel
- **Why?** Because there are no data dependencies across different values of *k*
- Increased locality due to the transformation as we decrease the reuse distance

Data Dependence Constraints



- If these constraints yield No Solution, then there is no data dependence across different values of *k* for the given pair of static accesses
- Check this for all possible pairs of static accesses (including self pairings)
- If the conditions are satisfied: <u>*the loop has 1 degree of parallelism*</u> (1 level of loop nest that can be parallelized)

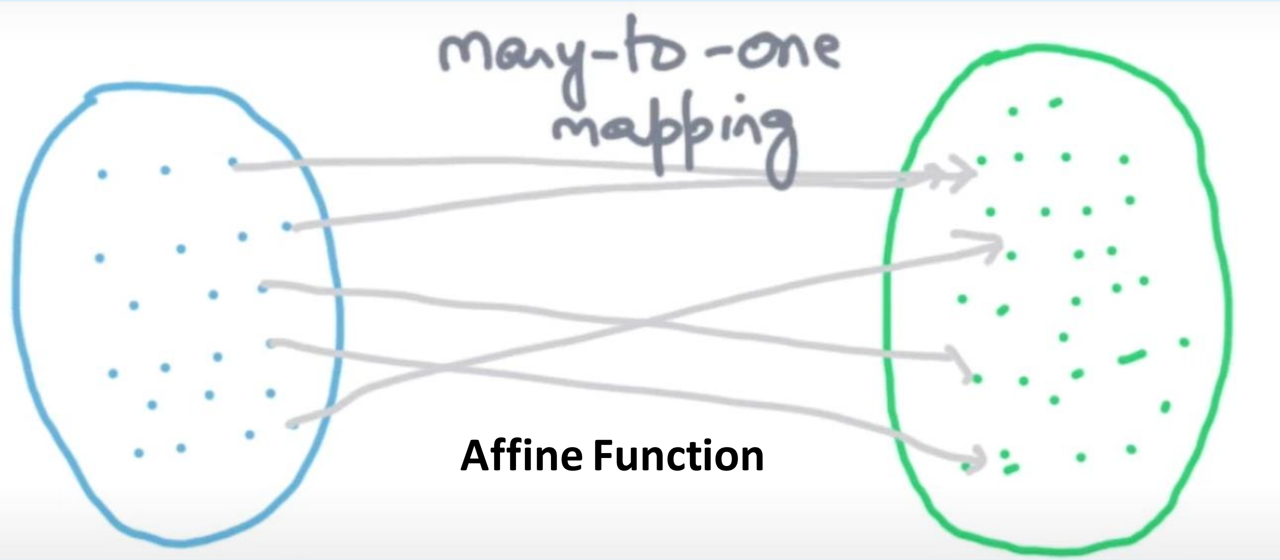
Degrees of parallelism

- A loop nest has *k* degrees of parallelism if it has, within the nest, *k* parallelizable *for* loops
- Can create $O(n^k)$ parallel virtual processors

```
par for(i = 0; k < n; i++)
  par for(j = 0; l < n; j++)
        for(k = 0; k < n; k++)
          A[i, j, k] = A[i, j, k - 1]
```

2 degrees of parallelism

Affine Space Partitions



Iteration-Space
l-dim

Processor-space
k-dim

- $l \geq k$: it is a many-to-one map, and a function (maps all the iterations)
- Need to use as many as (virtual) processors
- Partition: All the iterations in the partition are mapped to the same processor
- Constraint: This map needs to be an affine function

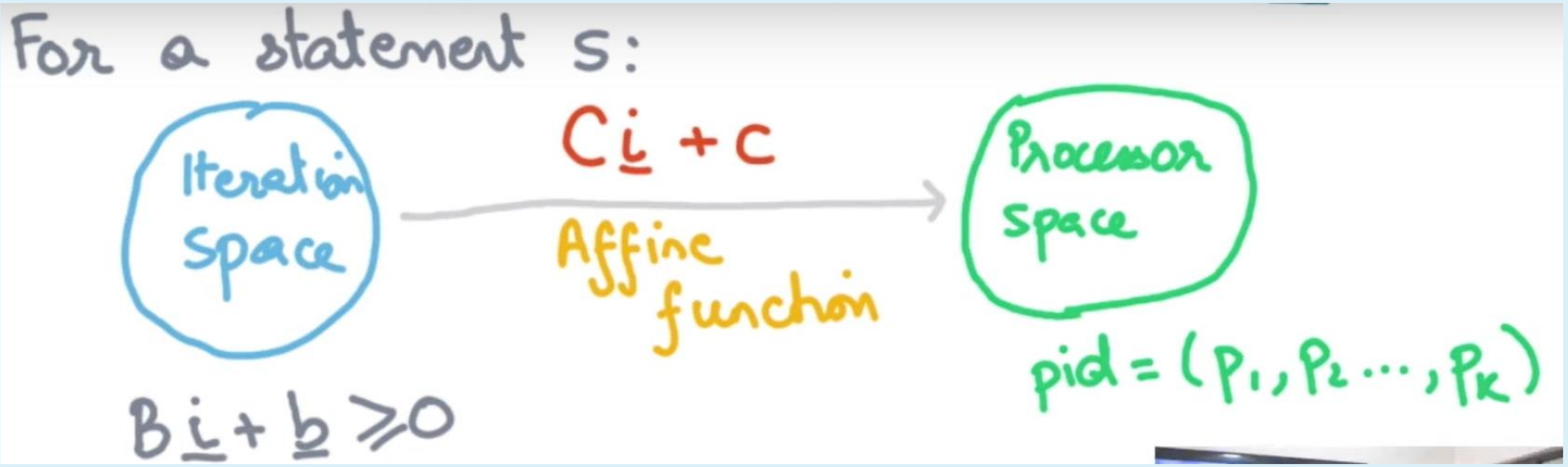
Affine Space Partitions

```
for (i=0; i<n; i++)
    A[i]=0;
    B[i]=1;
```

No data dependency

- Can have *O(2n)* virtual processors
- Our analysis should be able to exploit this
- Each 3AC statement (static access) is analyzed separately for maximum parallelism

Lec-142: Space Partition Constraints



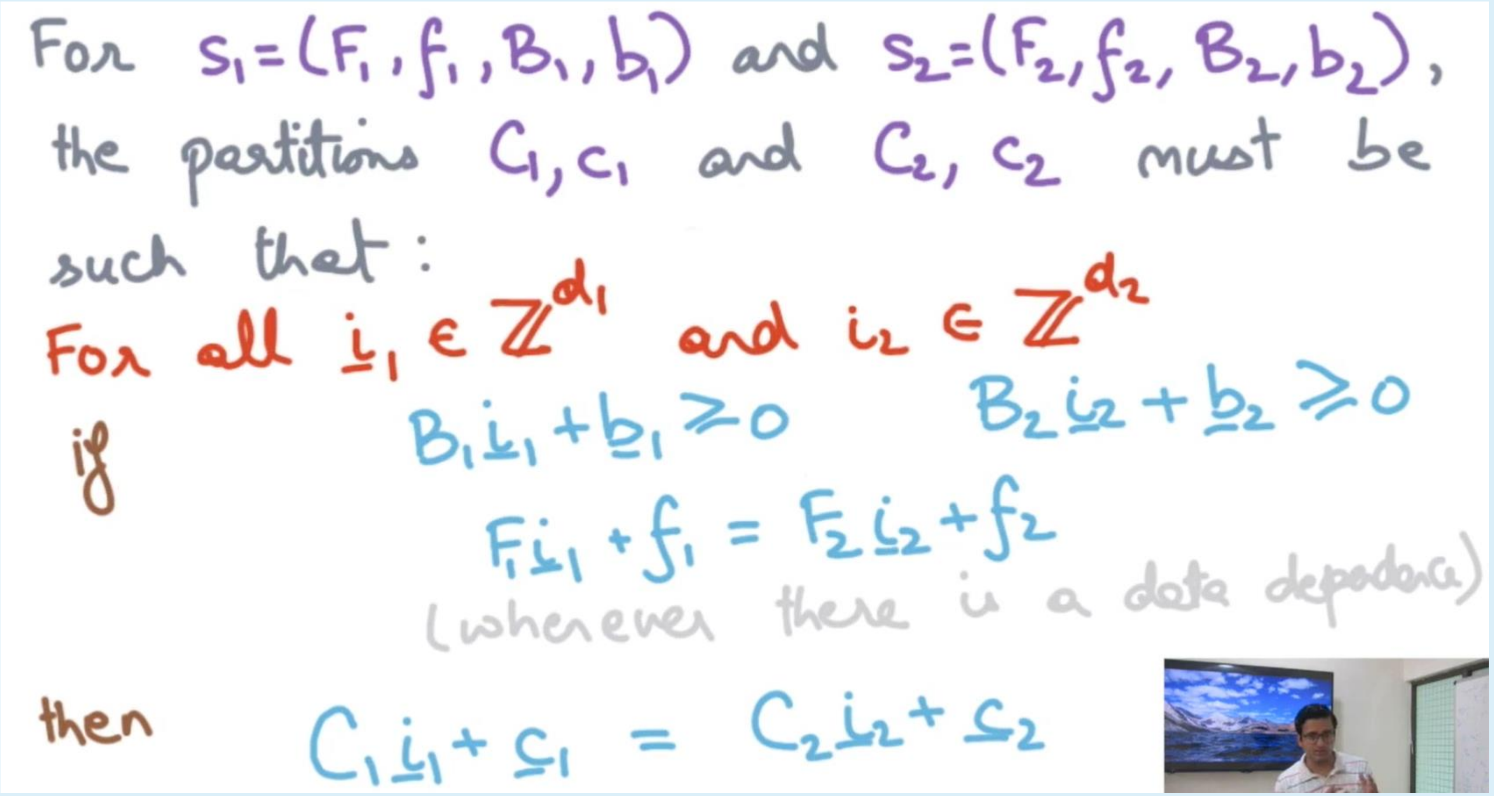
$$pid = Ci + c$$

- *(C, c)* are different for each statement *s*
- Variation: Could have a piecewise affine function instead of an affine function
- Piecewise affine functions can be solved using polyhedral analysis (potentially giving better results)
- Tradeoff: Performance vs Cost
- Restrict ourselves to Affine functions

Space Partition Constraints

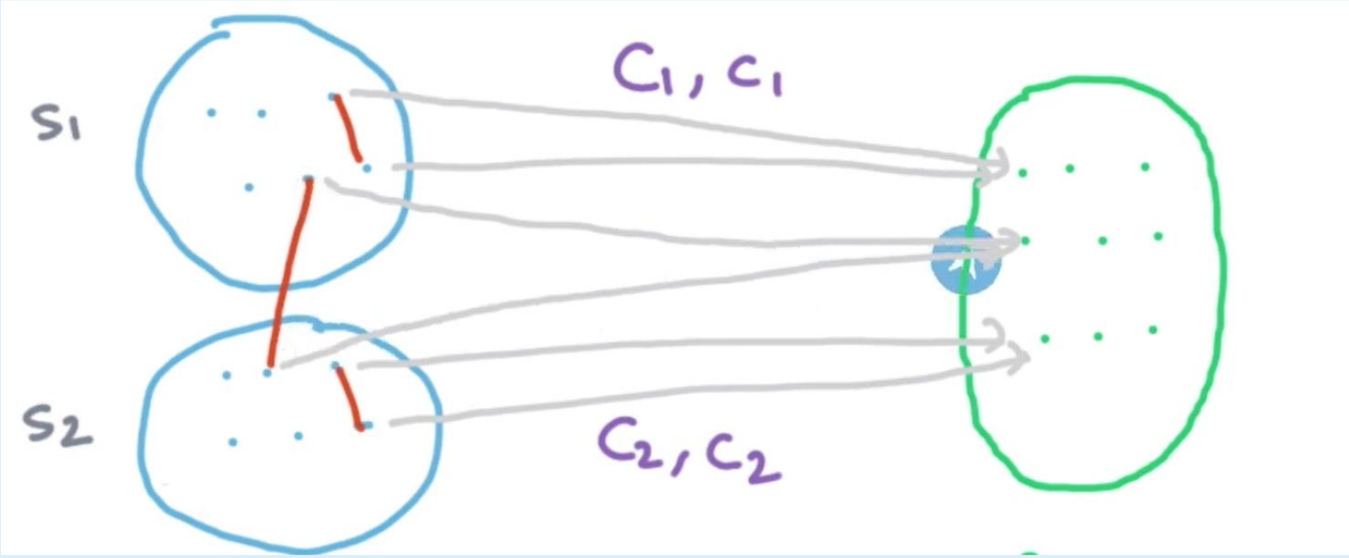
- Need to find a solution to *(C, c)* satisfying data dependency constraints
- Trivial Solution: $C = (0\ 0\ \ldots 0)$ and $c = (0)$
- Represents a zero-dimensional processor space
- Everything mapped to the same processor
- Valid solution but not very useful due to no parallelism
- Also need to maximize the processor space dimension / rank of *C*
- Affine Partition: a *(C, c)* solution for each statement *s* represents an **Affine partition**

Space Partition Constraints



- Data dependent iterations are mapped to the same processor
- Unknowns: C_1, c_1, C_2, c_2

Space Partition Constraints



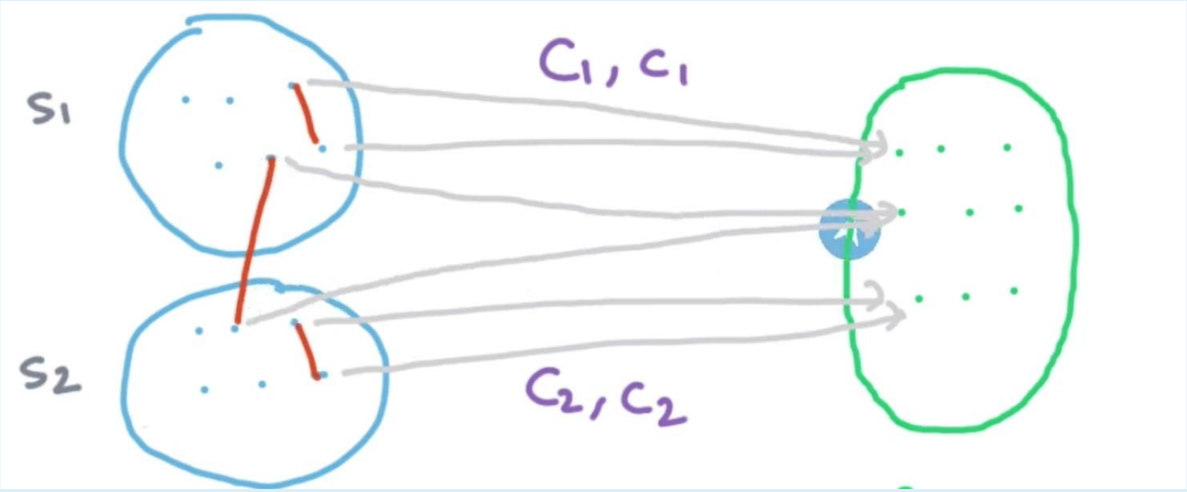
Iteration-Spaces

Processor-space

- Data dependence could be across the same statement or different statements
- Chose these unknowns such that the constraints are satisfied, and the ranks are maximized

Lec-143: Maximum Rank Affine Partition

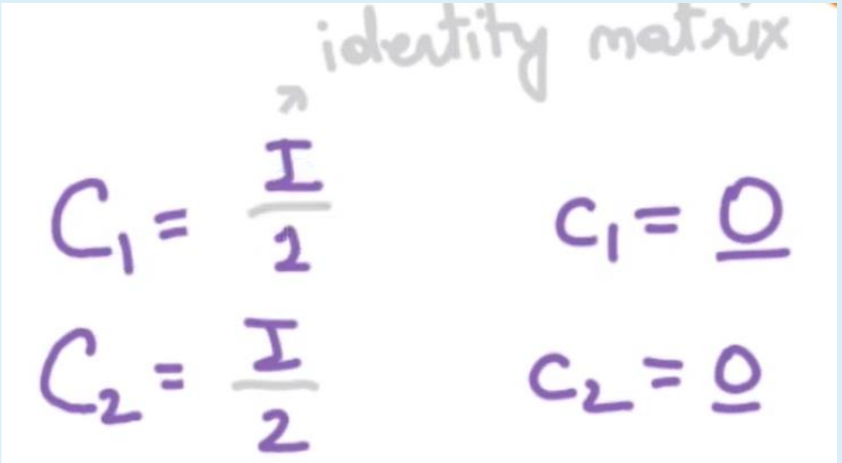
- Affine Partitions help us to argue about the processors, iterations and the data in a homogeneous way



- Trivial Solution: $C_1 = \underline{0}, c_1 = \underline{0}, C_2 = \underline{0}, c_2 = \underline{0}$
- But no parallelism

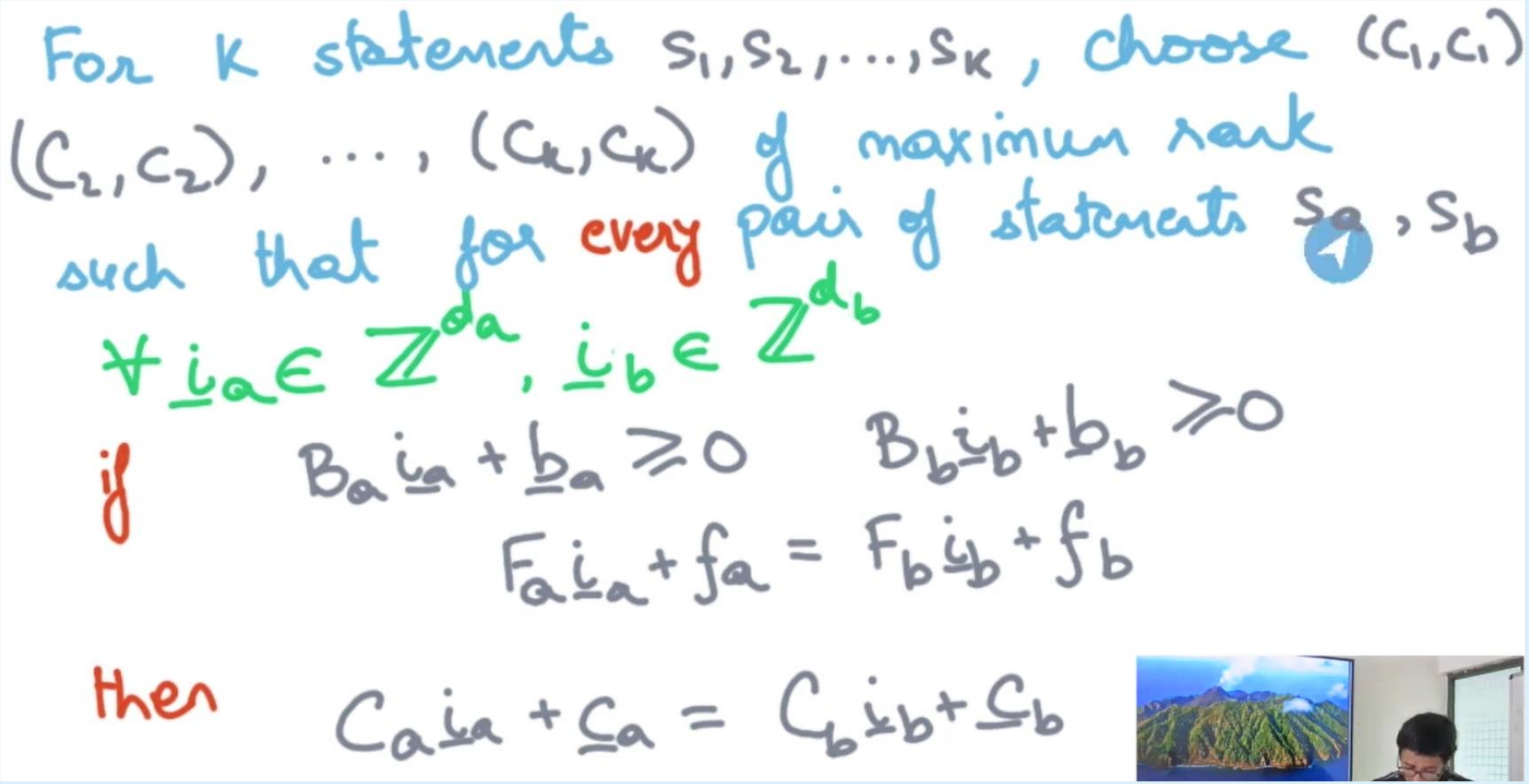
Maximum Rank Affine Partition

- Max Rank Solution:



- Desirable, but may not satisfy Data dependency constraints
- Interested in the Max Rank solution satisfying the data dependence constraints

Max Rank constraints



Space Partitioning Example

- Running Example:

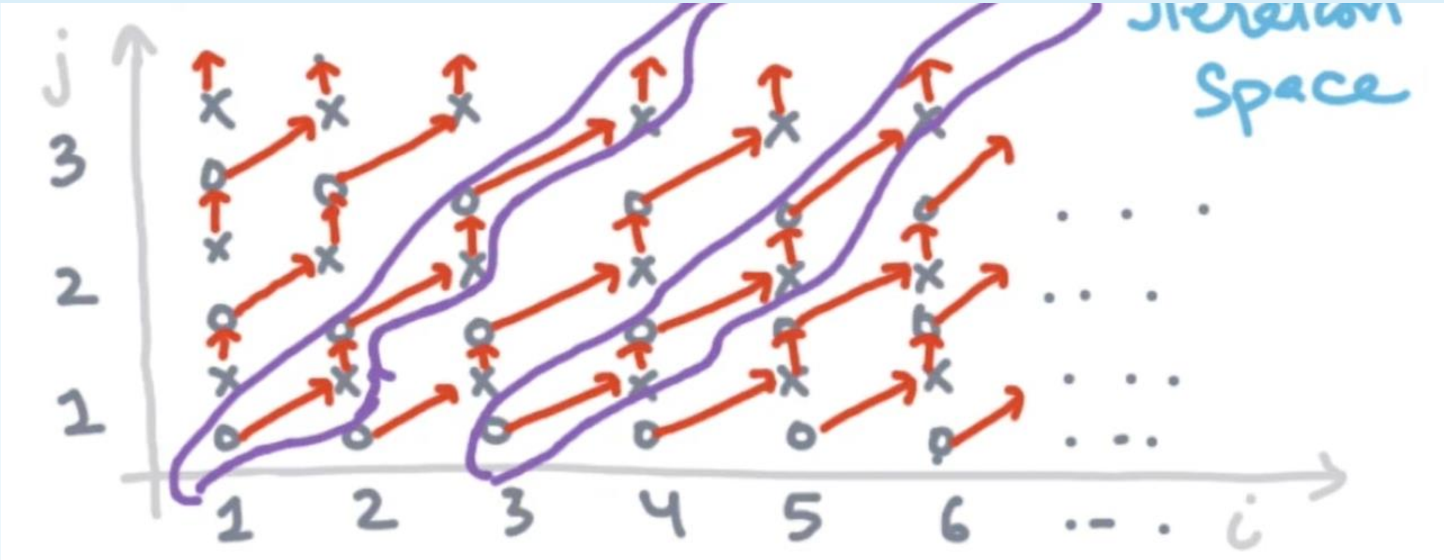
```
for (i=1; i<=100; i++)
    for (j=1 ; j<=100 ; j++)
        X[i,j] = X[i,j] + Y[i-1, j];
        Y[i,j] = Y[i,j] + X[i,j-1];
```

- Six Statements (one for each access); two writes
- Data dependences:
  - *X[i, j]* ↔ *X[i, j]* (both R/W)
  - *X[i, j]* ↔ *X[i, j-1]*
  - *Y[i, j]* ↔ *Y[i, j]* (both R/W)
  - *Y[i, j]* ↔ *Y[i-1, j]*

Space Partitioning Example

- *X[i, j]* with itself won't have any space partitioning because *[i, j]* is a full rank access
- *X[i, j] = X[i, j] + Y[i-1, j]:* we do have a data dependency, but in the same iteration
- No meaningful constraints
- We need only 2 affine functions, one for each C-statement (based on scalar dependencies)

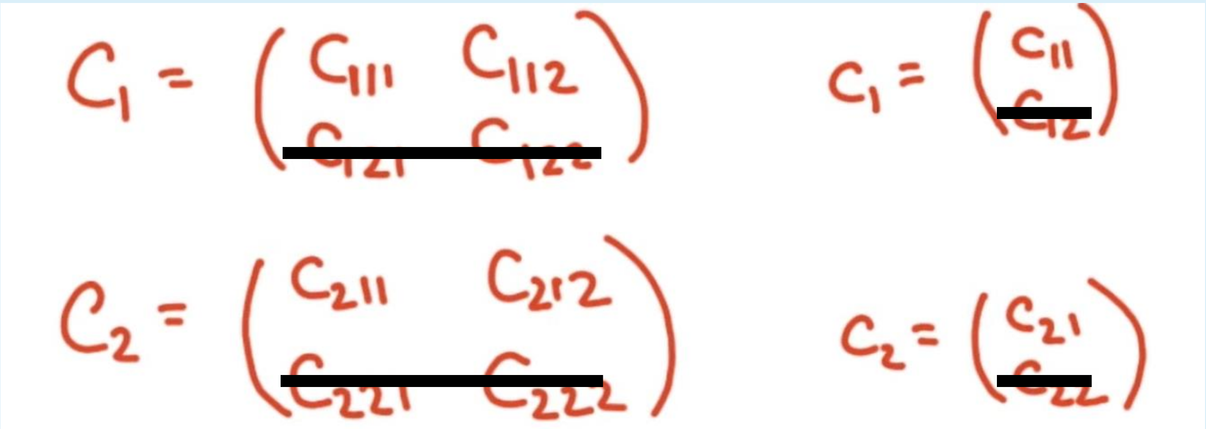
Space Partitioning Example



- All the dependent iterations should be mapped to the same processor
- Disjoint chains are formed
- Could map each of the chains to a separate processor
- Need to find C_1, c_1, C_2, c_2 such that each chain is mapped to the same processor
- Answer: 1-D mapping for each statement
  - $P_1 = i - j - 1$ for s_1
  - $P_2 = i - j$ for s_2

Lec-144: Space Partition Constraints Example

- Only data dependencies:
  - *X[i, j]* ↔ *X[i, j-1] (I)*
  - *Y[i-1, j]* ↔ *Y[i, j] (II)*
- 12 unknowns



- Could use previous knowledge that dimensionality of the processor space = 1 (C_1, C_2 have dependent rows)
- For now, assume that $C_{121} = C_{122} = 0 = C_{221} = C_{222}$

Space Partition Constraints Example

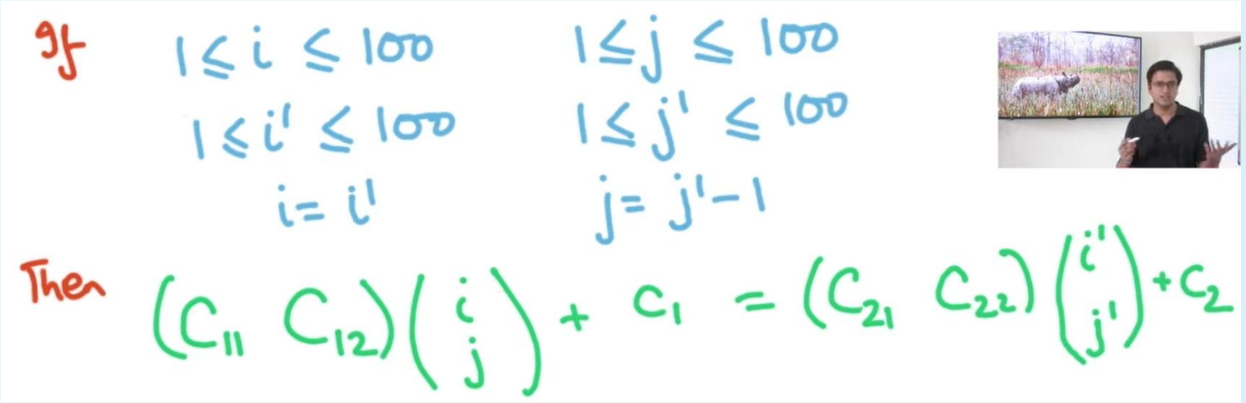
- New Problem:

$$C_1 = \begin{pmatrix} C_{11} & C_{12} \end{pmatrix} \qquad c_1 = \begin{pmatrix} c_1 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} C_{21} & C_{22} \end{pmatrix} \qquad c_2 = \begin{pmatrix} c_2 \end{pmatrix}$$

Space Partitioning Constraints

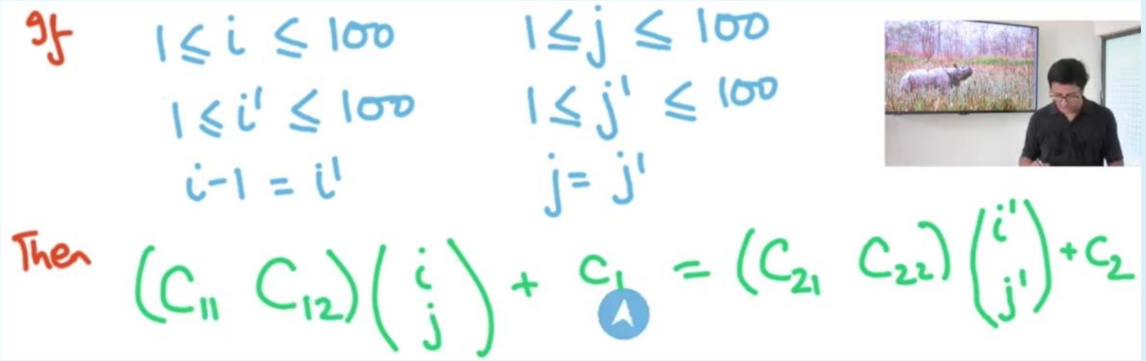
- For dependency *(I):*



- Possible Solution: $C_{11} = C_{21} = 1, C_{12} = C_{22} = 0, c_1 = c_2 = 0$
- Iteration *(i, j)* is mapped to processor i
- All conditions are satisfied (one of the possible solutions)
- But this is not the only constraint

Space Partitioning Constraints

- For dependency *(II):*



- Possible Solution: $C_{11} = C_{21} = 0, C_{12} = C_{22} = 1, c_1 = c_2 = 0$
- This solution satisfies the second dependency but not the first
- Previous solution does not satisfy this dependency
- Need a solution that satisfies both the constraints

Space Partitioning Example Solution

- Solution: $C_{11} = C_{21} = 1, C_{12} = C_{22} = -1, c_1 = -1, c_2 = 0$

$$i - j - 1 = i' - j'$$

- This holds for both the constraints

Lec-145: Solving Space Partition Constraints

$\forall\, i, j, i', j'$

if $1 \le i, i' \le 100$

$1 \le j, j' \le 100$

$i = i' \quad j = j' - 1$

then

$$\begin{pmatrix} C_{11} & C_{12} \end{pmatrix}\begin{pmatrix} i \\ j \end{pmatrix} + c_1 = \begin{pmatrix} C_{21} & C_{22} \end{pmatrix}\begin{pmatrix} i' \\ j' \end{pmatrix} + c_2$$

$\forall\, i, j, i', j'$

if $1 \le i, i' \le 100$

$1 \le j, j' \le 100$

$i - 1 = i' \quad j = j'$

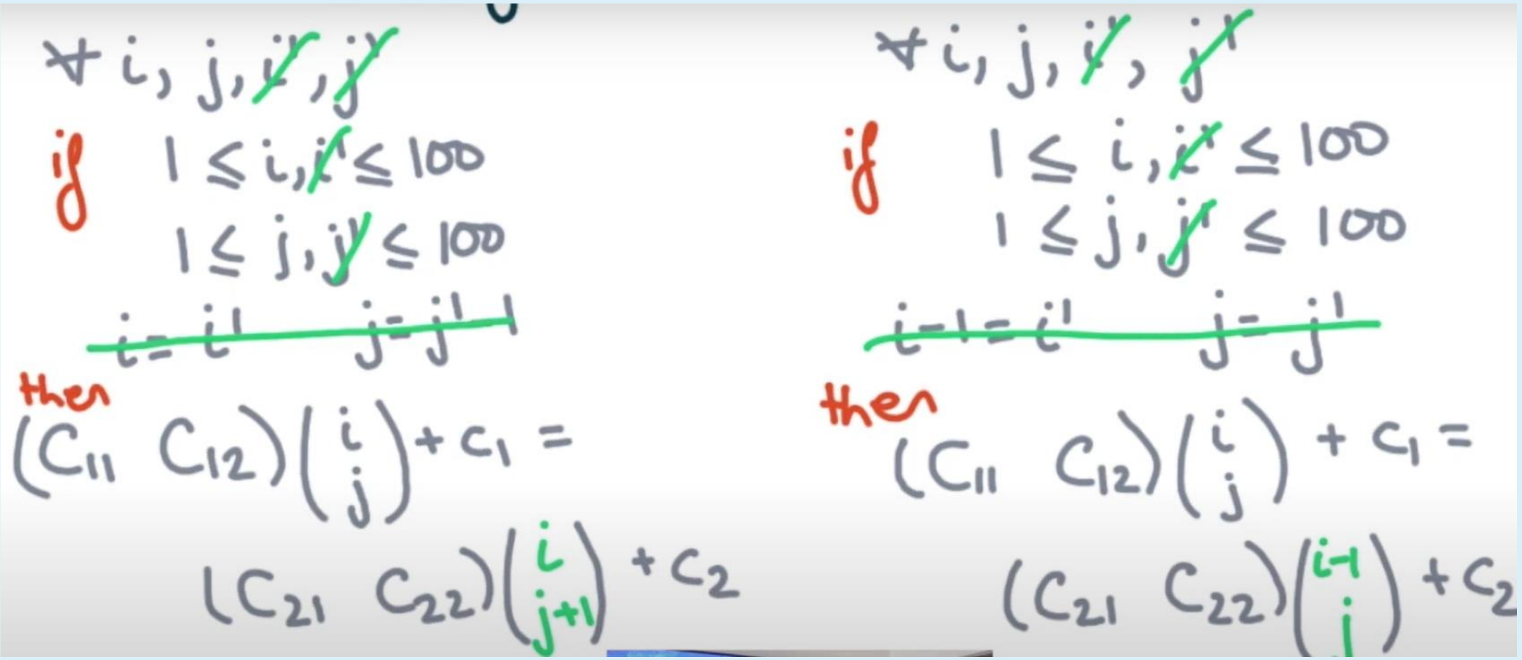
then

$$\begin{pmatrix} C_{11} & C_{12} \end{pmatrix}\begin{pmatrix} i \\ j \end{pmatrix} + c_1 = \begin{pmatrix} C_{21} & C_{22} \end{pmatrix}\begin{pmatrix} i' \\ j' \end{pmatrix} + c_2$$

- Both these constraints must be satisfied
- *i, j, i', j'* are not unknowns

Solving Space Partition Constraints

- Use gaussian elimination to get rid of some variables
- Use the constrains to eliminate *i’, j’*



Solving Space Partition Constraints

- Rewrite the equations:

$$\begin{pmatrix} (C_{11} - C_{21}) & (C_{12} - C_{22}) \end{pmatrix} \begin{pmatrix} i \\ j \end{pmatrix} + c_1 - c_{22} - c_2 = 0$$

$$\begin{pmatrix} (C_{11} - C_{21}) & (C_{12} - C_{22}) \end{pmatrix} \begin{pmatrix} i \\ j \end{pmatrix} + c_1 + c_{21} - c_2 = 0$$

Solving Space Partition Constraints

- Overapproximate the behavior of iteration variables for these constraints
- Assume that the equations hold for all real values of *i* , *j*
- Which means that the coefficient of *i, j* and the constant term are zero

$$C_{11} = C_{21}$$
$$C_{12} = C_{22}$$
$$c_1 = c_2 + C_{22}$$
$$c_1 = c_2 - C_{21}$$

Solving Space Partition Constraints

- On solving, we get:

$$-C_{12} = -C_{22} = C_{21} = C_{11} = C$$
$$c_1 = c_2 - C$$

- Actual constant values don't matter because they only shift the space of processor IDs
- WLOG, pick $C = 1$, $c_2 = 0$
- So, we get the same solution as before
  - $P_1 = i - j - 1$
  - $P_2 = i - j$
- Remaining: we need to make sure that all the iterations mapped to the same processor preserve the relative order of execution of these iterations

Thank You!

- Jai Arora